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LETTER TO THE EDITOR

Parity measurements, decoherence and spiky Wigner functions

A M Ozorio de Almeida and O Brodier

Centro Brasileiro de Pesquisas Fisicas, Rua Xavier Sigaud 150, 22290-180, Rio de Janeiro, RJ, Brazil

E-mail: ozorio@cbpf.br and brodier@cbpf.br

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Abstract
Notwithstanding radical conceptual differences between classical and quantum mechanics, it is usually assumed that physical measurements concern observables common to both theories. Not so with the eigenvalues (± 1) of the parity operator. The effect of such a measurement on a mixture of even and odd states of the harmonic oscillator is akin to separating at a single stroke a pair of shuffled card decks: the result is a set of definite parity, though otherwise mixed. Here we derive the general form of a parity collapsed state, whether pure or mixed. The signature of positive or negative parity is a corresponding spike in the Wigner function which is sharpened by decoherence. We conjecture that states with pure parity always have negative values in their Wigner functions.

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(Some figures in this article are in colour only in the electronic version)

The parity operator, $\widehat{R}_{\mathbf{0}} = (-1)^{\widehat{n}}$, is an observable that is currently measured in quantum optics [1–3]. Here $\widehat{n} = \widehat{H}/\hbar$, where $\hbar$ is the Planck constant and $\widehat{H}$ is the Hamiltonian of the harmonic oscillator with unit frequency. Thus the number (or Fock) states of the harmonic oscillator are specific examples of parity eigenstates, but the parity operator, or the reflection with respect to any other phase space point, $\widehat{R}_{\mathbf{x}}$, can be applied to arbitrary states. The important point is that parity is a pure quantum property, which can be generated by a measurement even on a mixed, classical-like state resulting from decoherent evolution [4, 5]. In such a case, quantitative measures of decoherence, such as linear, or Von Neumann entropy, will still point out the lack of purity after the collapse onto a definite parity. Focussing on the parity is a promising strategy for dealing with decoherence of a quantum system, for, if the loss of a quantum in an orthogonal basis switches the parity, a second loss restores it. This property can be used for atomic interfrometers [1] and has also been suggested for stabilizing quantum computers [6] and for teleportation protocols [7].

Instead of depending on an infinite basis in Hilbert space to describe mixed, parity reduced states, it is much more sensible to rely on the Wigner function, $W(\mathbf{x})$ [8], which is proportional to the expectation of $\widehat{R}_{\mathbf{x}}$ [9, 10]. The quantum operator, $\widehat{R}_{\mathbf{x}}$, corresponds to a (classical) reflection through the phase space point $\mathbf{x} = (p, q)$, i.e. other points $\mathbf{x}' \to 2\mathbf{x} - \mathbf{x}'$. It is possible to specify $\widehat{R}_{\mathbf{x}}$ by a superposition of projection operators [11], $|q\rangle\langle q'|$:

$$\widehat{R}_{\mathbf{x}} = \frac{1}{2}\int \mathrm{d}q' \left| q - \frac{q'}{2} \right\rangle \exp\left(-\mathrm{i}\frac{pq'}{\hbar}\right) \left\langle q + \frac{q'}{2} \right|. \tag{1}$$

From this we obtain the well-known definition of the Weyl symbol for an arbitrary operator, $\widehat{A}$, as [12–14]

$$A(\mathbf{x}) = 2\mathrm{Tr}\, \widehat{R}_{\mathbf{x}}\widehat{A} = \int \mathrm{d}q' \left\langle q + \frac{q'}{2} \right| \widehat{A} \left| q - \frac{q'}{2} \right\rangle \exp\left(-\mathrm{i}\frac{pq'}{\hbar}\right). \tag{2}$$

Here, 'Tr' denotes the sum over all eigenvalues of an operator. In the case of the density operator, $\widehat{\rho}$, it is conveniently normalized to obtain the Wigner function [11], $W(\mathbf{x}) = \rho(\mathbf{x})/2\pi\hbar$. The direct measurement of the Wigner function results from counting the relative proportions of the eigenvalues ± 1 for repeated parity measurements [3]. The density matrix in the position representation follows from the inverse of the Fourier transform (2).

The result of a parity measurement for reflection through a point $\mathbf{X}$ on a quantum system described by the density operator $\widehat{\rho}$ must be one of the alternatives

$$\begin{cases} \widehat{\rho}^{\mathbf{X}}_{+} = \frac{\widehat{P}^{\mathbf{X}}_{+}\widehat{\rho}\,\widehat{P}^{\mathbf{X}}_{+}}{\mathrm{Tr}\,\widehat{\rho}\,\widehat{P}^{\mathbf{X}}_{+}} \\ \widehat{\rho}^{\mathbf{X}}_{-} = \frac{\widehat{P}^{\mathbf{X}}_{-}\widehat{\rho}\,\widehat{P}^{\mathbf{X}}_{-}}{\mathrm{Tr}\,\widehat{\rho}\,\widehat{P}^{\mathbf{X}}_{-}} \end{cases} \tag{3}$$

allowed by the standard quantum theory [15], where the orthogonal projection operators for each parity are [11]

$$\widehat{P}^{\mathbf{X}}_{\pm} = \tfrac{1}{2}(1 \pm \pi\hbar\widehat{R}_{\mathbf{X}}). \tag{4}$$

From (2) and (4) we immediately obtain

$$\mathrm{Tr}\,\widehat{\rho}\,\widehat{P}^{\mathbf{X}}_{\pm} = \tfrac{1}{2}(1 \pm \pi\hbar W(\mathbf{x})). \tag{5}$$

The full Wigner function corresponding to $\widehat{\rho}^{\mathbf{X}}_{\pm}$ depends on the symplectic matrix

$$\mathbf{J} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \tag{6}$$

and the Fourier transform

$$\widetilde{W}(\xi) = \frac{1}{2\pi\hbar}\int \mathrm{d}\mathbf{x}\, W(\mathbf{x}) \exp\left(\frac{\mathrm{i}}{\hbar}\mathbf{x}\cdot\mathbf{J}\xi\right) \tag{7}$$

which is itself a bona fide representation of the density matrix, known as the chord function [14], or the characteristic function in quantum optics. Then the Wigner functions corresponding to the projected densities (3) are

$$W^{\mathbf{X}}_{\pm}(\mathbf{x}) = \frac{1}{2}\frac{W(\mathbf{x}) + W(2\mathbf{X}-\mathbf{x}) \pm 4\Re\widetilde{W}(2(\mathbf{x}-\mathbf{X}))\exp\left(-\frac{2\mathrm{i}}{\hbar}\mathbf{x}\cdot\mathbf{J}\mathbf{X}\right)}{1 \pm \pi\hbar W(\mathbf{X})} \tag{8}$$

where $\Re$ denotes the real part of a number. This formula generalizes the specific formula for $W^{\mathbf{0}}_{\pm}(\mathbf{x})$ in the case of circular symmetry [1].

The derivation of (8) is straightforward if one combines the reflection operators $\widehat{R}_{\mathbf{x}}$ with the translation operators,

$$\widehat{T}_{\xi} = \exp\left(\frac{\mathrm{i}}{\hbar}\widehat{\mathbf{x}}\cdot\mathbf{J}\xi\right) = \exp\left(\frac{\mathrm{i}}{\hbar}(\xi_p\widehat{q} - \xi_q\widehat{p})\right) \tag{9}$$

to form a quantum version [14] of the affine group of the translations and reflections [16] in phase space:

$$\widehat{T}_{\xi_1}\widehat{T}_{\xi_2} = \widehat{T}_{\xi_1+\xi_2}\exp\left(-\frac{\mathrm{i}}{2\hbar}\xi_1\cdot\mathbf{J}\xi_2\right) \qquad \widehat{T}_{\xi}\widehat{R}_{\mathbf{x}} = \widehat{R}_{\mathbf{x}+\frac{\xi}{2}}\exp\left(-\frac{\mathrm{i}}{\hbar}\mathbf{x}\cdot\mathbf{J}\xi\right)$$
$$\widehat{R}_{\mathbf{x}}\widehat{T}_{\xi} = \widehat{R}_{\mathbf{x}-\frac{\xi}{2}}\exp\left(-\frac{\mathrm{i}}{\hbar}\mathbf{x}\cdot\mathbf{J}\xi\right) \qquad \widehat{R}_{\mathbf{x}_1}\widehat{R}_{\mathbf{x}_2} = \widehat{T}_{2(\mathbf{x}_1-\mathbf{x}_2)}\exp\left(\frac{2\mathrm{i}}{\hbar}\mathbf{x}_1\cdot\mathbf{J}\mathbf{x}_2\right). \tag{10}$$

Thus, the Weyl symbol corresponding to $\widehat{P}^{\mathbf{X}}_{\pm}\widehat{\rho}\,\widehat{P}^{\mathbf{X}}_{\pm}$ is

$$\begin{aligned}\left[\widehat{P}^{\mathbf{X}}_{\pm}\widehat{\rho}\,\widehat{P}^{\mathbf{X}}_{\pm}\right](\mathbf{x}) &= \frac{1}{2}(\mathrm{Tr}\,\widehat{R}_{\mathbf{x}}\widehat{\rho} \pm \mathrm{Tr}\,\widehat{R}_{\mathbf{x}}\widehat{R}_{\mathbf{X}}\widehat{\rho} \pm \mathrm{Tr}\,\widehat{R}_{\mathbf{x}}\widehat{\rho}\,\widehat{R}_{\mathbf{X}} + \mathrm{Tr}\,\widehat{R}_{\mathbf{x}}\widehat{R}_{\mathbf{X}}\widehat{\rho}\,\widehat{R}_{\mathbf{X}})\\ &= \frac{1}{2}\left(\mathrm{Tr}\,\widehat{R}_{\mathbf{x}}\widehat{\rho} + \mathrm{Tr}\,\widehat{R}_{2\mathbf{X}-\mathbf{x}}\widehat{\rho} \pm 2\Re\exp\left(-\frac{2\mathrm{i}}{\hbar}\mathbf{x}\cdot\mathbf{J}\mathbf{X}\right)\mathrm{Tr}\,\widehat{T}_{2(\mathbf{X}-\mathbf{x})}\widehat{\rho}\right).\end{aligned} \tag{11}$$

Therefore the first two terms lead directly to Wigner functions and we obtain (8) by using the alternative definition of the chord representation [14],

$$\widetilde{A}(\xi) = \mathrm{Tr}\,\widehat{T}_{-\xi}\widehat{A}. \tag{12}$$

The complete knowledge of the Wigner function can be translated into any other representation of the density matrix, but we shall now show that the special features of parity reduced states are clearly manifest in the Wigner–Weyl representation. It is well known that the Wigner function of most pure states displays narrow negative oscillations. If the quantum system evolves in contact with the external environment, decoherence [4, 5] smoothes the Wigner function and eventually erases the negative fringes. The threshold time for complete positivity [17, 18] of the Wigner function is independent of the initial pure state, within the Markovian approximation and the further assumptions of linear coupling and quadratic Hamiltonian [18]. In the following simple illustrations of this scenario we shall verify that a measurement of the parity operator generates a central spike of maximum modulus [19], $W(\mathbf{0}) = \pm(\pi\hbar)^{-1}$, on the previously smoothed Wigner function. Furthermore, a weaker pattern of fringes reemerges, resembling those of pure states. Quantitative measures still indicate overall decoherence, confirmed by the coarse-graining of $W(\mathbf{x})$, far from the reflection centre. Even so, the positivity threshold for the further Markovian evolution of an odd state, $W_-(\mathbf{x})$, is the same as for a pure state, generally exceeding the time for ordinary mixed states to lose their negative regions. The sharp spike of $W_\pm(\mathbf{x})$ signals the full recovery of quantum parity as a consequence of its experimental measurement. It should be noted that the following calculations are exact, with no basis truncations in spite of the decoherence. Thus the Wigner function fully reveals the hybrid structure of parity reduced mixed states.

As a first example, consider the Wigner function corresponding to a pure coherent state [15, 10], $|\mathbf{Y}\rangle$, with $\langle q\rangle = Q$ and $\langle p\rangle = P$. It is well known that this Wigner function is

$$W_{\mathbf{Y}}(\mathbf{x}) = \frac{1}{\pi\hbar}\exp\left(-\frac{(\mathbf{x}-\mathbf{Y})^2}{\hbar}\right) = \frac{1}{\pi\hbar}\exp\left(-\frac{(p-P)^2+(q-Q)^2}{\hbar}\right) \tag{13}$$

i.e. just a minimum uncertainty Gaussian. The measurement of parity with respect to the origin [20] produces one of the alternatives allowed by (8), which can be interpreted as pure Wigner functions corresponding to the sum or the difference of the coherent states $|\mathbf{Y}\rangle$ and $|-\mathbf{Y}\rangle$:

$$W^{\mathbf{0}}_{\pm}(\mathbf{x}) = \frac{1}{2\pi\hbar}\frac{\exp\left(-\frac{(\mathbf{x}-\mathbf{Y})^2}{\hbar}\right) + \exp\left(-\frac{(\mathbf{x}+\mathbf{Y})^2}{\hbar}\right) \pm 2\exp\left(-\frac{\mathbf{x}^2}{\hbar}\right)\cos\left(\frac{2\mathbf{x}\cdot\mathbf{J}\mathbf{Y}}{\hbar}\right)}{1\pm\pi\hbar\exp\left(-\frac{\mathbf{Y}^2}{\hbar}\right)} \tag{14}$$

For sufficiently large components of $\mathbf{Y}$, $W_{\mathbf{Y}}(\mathbf{0})$ is very small, so we have nearly the same probability of obtaining the state $|+\rangle$, corresponding to $W^{\mathbf{0}}_{+}(\mathbf{x})$, as the state $|-\rangle$. Both these

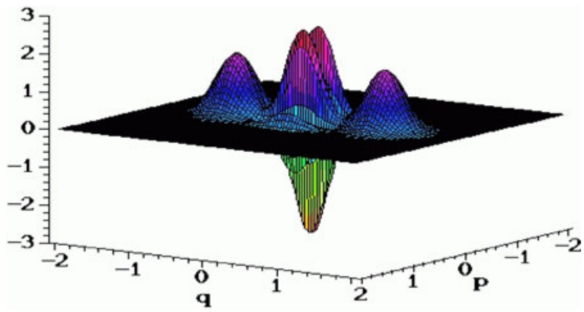

Figure 1. Wigner function of an odd superposition of two coherent states in units where $\hbar = 0.1$. The horizontal plane is the phase space, $\mathbf{x} = (p, q)$.

projected Wigner functions resolve into three separate Gaussians. Those centred on $\pm\mathbf{Y}$ are smooth, whereas the Gaussian at the origin is modulated by fringes. These states are sometimes referred to as 'Schrödinger cat states' and it is easily verified that $W^{\mathbf{0}}_{\pm}(\mathbf{0}) = \pm(\pi\hbar)^{-1}$. Figure 1 presents the familiar form of $W^{\mathbf{0}}_{-}(\mathbf{x})$. The 'subplanckian' scale of the fine oscillations near the origin is taken to be a sure sign of quantum coherence [21].

Already in this simple example, we met the strangeness peculiar to parity measurements. The linearity of quantum mechanics allows us to describe the states $|+\rangle$ and $|-\rangle$ as alternative superpositions of the states $|\mathbf{Y}\rangle$ and $|-\mathbf{Y}\rangle$. It may seem perverse, but we can equally describe the latter classical-like states as particular superpositions of the Schrödinger cats, $|+\rangle$ and $|-\rangle$. Indeed, an ideal parity measurement enforces this unintuitive interpretation. Since the projections of the Wigner function provide real probabilities, it follows that, after the parity measurement, the position $-Q$ is just as likely as Q and, likewise, the momenta $-P$ and P are equally probable, even though the negative options would be most unprobable in the initial state.

If the system is not completely isolated from the external environment, even an initially pure state evolves into a mixture, i.e. the density operator develops into a probability distribution over pure state densities. A simple example is a system in which we neglect the action of an internal Hamiltonian while allowing for linear coupling with the environment through the position $\widehat{q}$ and the momentum $\widehat{p}$. The solution of the Fokker–Planck equation [4, 18] that determines the evolution of the Wigner function is

$$W(\mathbf{x}, t) = \frac{1}{2\pi\hbar}\int \mathrm{d}\mathbf{y}\, W(\mathbf{y}-\mathbf{x}, 0)\exp\left(-\frac{\mathbf{y}^2}{2\hbar c^2 t}\right). \tag{15}$$

This effect of the environment that progressively coarse-grains an initial pure state is more general than would appear in our simple model. Internal motion and dissipative coupling to the environment can also be included [18]. Proceeding, though, with the evolution (15) for the initial Schrödinger cat state $W(\mathbf{x}, 0) = W^{\mathbf{0}}_{-}(\mathbf{x})$, we obtain

$$W(\mathbf{x}, t) = \frac{N}{\pi\hbar(2c^2t+1)}\left[\exp\left(-\frac{(\mathbf{x}-\mathbf{Y})^2}{\hbar(2c^2t+1)}\right) + \exp\left(-\frac{(\mathbf{x}+\mathbf{Y})^2}{\hbar(2c^2t+1)}\right)\right.$$
$$\left. - 2\exp\left(-\frac{\mathbf{x}^2}{\hbar(2c^2t+1)}\right)\exp\left(-\frac{2c^2t\mathbf{Y}^2}{\hbar(2c^2t+1)}\right)\cos\left(\frac{2\mathbf{x}\cdot\mathbf{J}\mathbf{Y}}{\hbar(2c^2t+1)}\right)\right] \tag{16}$$

with $N^{-1} = 2(1-\exp(-\mathbf{Y}^2/\hbar))$. Thus, the positivity threshold is $t_0 = 1/(2c^2)$ in this case. The full Wigner function $W(\mathbf{x}, t_0)$ is shown in figure 2. One should be aware that the symmetry of $W(\mathbf{x}, t_0)$ as regards to $\mathbf{0}$ has nothing to do with the parity of the mixture of states

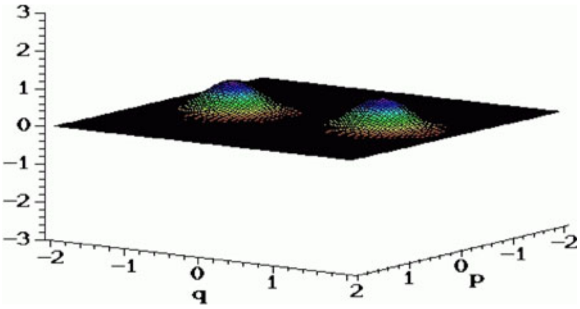

Figure 2. Decoherent evolution of the Wigner function in figure 1 at the positivity threshold.

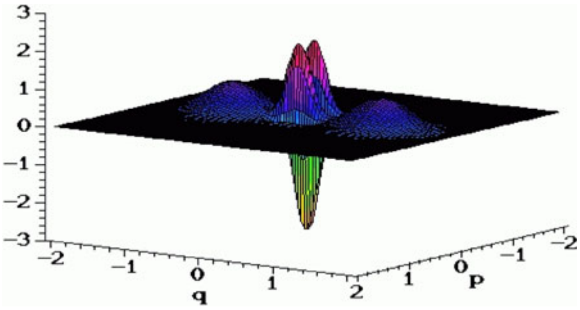

Figure 3. Wigner function after an odd measurement carried out on the mixture represented in figure 2.

it represents. In fact, one has $W(\mathbf{0}, t_0) = 0$, which shows that the probabilities of an even or odd parity measurement are actually equal.

The result (8) of a further ideal odd parity measurement on the mixed Wigner function $W(\mathbf{x}, t_0) = 0$ is displayed in figure 3. Again the value of the Wigner function is brought down to its minimum $-(\pi\hbar)^{-1}$, but the neighbouring interference fringes are only partly regenerated by the measurement. Thus, the hybrid nature of the state, which is pure only as concerns parity with respect to $\widehat{R}_{\mathbf{0}}$, is graphically exhibited by its Wigner function.

Allowing this spiky state to interact with the environment as before, we immediately verify that the corresponding Wigner function becomes positive again as soon as the further interval t_0 has passed, just as if it were a pure state. This follows from a simple extension of our previous arguments [18]. Ordinary mixed states lose the negative regions of their Wigner function before pure states, but odd parity mixtures must await for the pure state threshold. This depends only on the parameters of the internal quadratic Hamiltonian and of the linear coupling to the environment. If the initial state $|-\rangle$ evolves for a longer time in contact with the outside environment, the two mounds in figure 2 erode even further and eventually interpenetrate. Figure 4 shows a profile of the sharp spike that is superimposed on this smooth classical background by a positive parity measurement. Note the small negative ripples, which are tell-tale signs of quantum coherence.

So far, all our computations support the conjecture that $W_+(\mathbf{x})$, as well as $W_-(\mathbf{x})$, always takes on negative values, no matter how far decoherence has proceeded prior to the parity measurement.

We have derived the general form of parity reduced mixed states and shown that their hybrid property, that is intertwining between classical coarse graining and fine nonlocal

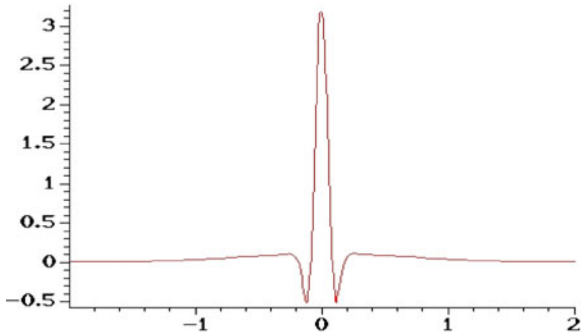

Figure 4. Profile along a diagonal direction in the phase space of the Wigner function reduced by an even measurement far beyond the positivity threshold.

quantum information coded into a selection rule, is suitably resolved by the corresponding Wigner function. The latter, indeed, distil all the quantumness of the mixture into a local spike in the otherwise smoothed phase space geography. This quantum strangeness has no counterpart in classical waves and their Wigner functions [22]. Any well tuned ensemble of clarinets is capable of producing sound waves where the odd harmonics of the fundamental note are missing. It should be pointed out, however, that there is no relation of such classical standing waves and their harmonics, with the odd or even number of quantized photons in an optical cavity, which are all of the same frequency. All the same, there is a sense in which the manipulation of ideal parity measurements imposes the waviness of quantum matter. For example, if the parity of a mixed state of photons in a cavity is measured and immediately afterwards a photon escapes and is detected, the main effect should be the reversal of the sign of the central spike [19]. To what extent real laboratory experiments will be able to evince the full features of spiky Wigner functions remains to be seen. The initial experiments in quantum optics involving single atom masers [3] are impressive, but, so far, they have been dedicated to the measurement of the Wigner function, rather than to the production of a new kind of quantum state.

All the formulae in this letter have been presented for systems with a single degree of freedom, but they are easily generalized by extending the matrix **J** to higher dimensions and by suitably altering the powers of $2\pi\hbar$.

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